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R^4 terms in 11 dimensions and conformal anomaly of (2,0) theory

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Abstract

Using AdS$_7$/CFT$_6$ correspondence we compute a subleading $O(N)$ term in the scale anomaly of (2,0) theory describing N coincident M5 branes. While the leading $O(N^3)$ contribution to the anomaly is determined by the value of the supergravity action, the $O(N)$ contribution comes from a particular R^4 term (8-d Euler density invariant) in the 11-dimensional effective action. This R^4 term is argued to be part of the same superinvariant as the P-odd $\mathcal{C}_3 R^4$ term known to produce $O(N)$ contribution to the R-symmetry anomaly of (2,0) theory. The known results for R-anomaly suggest that the total scale anomaly extrapolated to N=1 should be the same as the anomaly of a single free (2,0) tensor multiplet. A proposed explanation of this agreement is that the coefficient $4N^3$ in the anomaly (which was found previously to be also the ratio of the 2-point and 3-point graviton correlators in the (2,0) theory and in the free tensor multiplet theory) is shifted to $4N^3 - 3N$.

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1. Introduction

Two known maximally (2,0) supersymmetric conformal field theories in 6 dimensions are the free tensor multiplet theory describing low energy dynamics of a single M5 brane, and still largely mysterious interacting (2,0) conformal theory describing N coincident M5 branes. A way to study the latter theory is provided by its conjectured duality [1] to M-theory (or, for large N, 11-d supergravity corrected by higher derivative terms) on $AdS_7 \times S^4$ background.

Comparison of the 2-point and 3-point correlators of the stress tensor of (2,0) theory as predicted by the $AdS_7 \times S^4$ supergravity [2,3] to those in the free tensor multiplet theory shows [4,5,6] that they differ only by the overall coefficient $4N^3$.[1] The remarkable coefficient $4N^3$ was originally found in [5] in the comparison of the M5 brane world volume theory and the $D = 11$ supergravity expressions for the absorption cross-sections of longitudinally polarized gravitons by N coincident M5 branes. The same coefficient $4N^3$ appears also as the ratio of the scale anomalies (or Weyl-invariant parts of conformal anomalies) of the interacting (2,0) theory [8] and free theory of a single tensor multiplet [9].

The reason why the coefficient $4N^3$ was puzzling in [5] was analogy with the $d = 4$ case: a similar comparison of the gravitational and world-volume absorption cross-sections in the case of D3-branes [10,5] led to the ratio N^2, which is equal to 1 for $N = 1$. This agreement in the $d = 4$ case was later understood [6] as being a consequence of nonrenormalization of the conformal anomaly and thus of the 2-point stress tensor correlator in $\mathcal{N} = 4$ SYM theory. The analogy between the $d = 4$ and $d = 6$ cases should not, of course, be taken too seriously, given that the (2,0) theory should have a different structure than SYM theory, being an interacting conformal fixed point without a free coupling parameter.

Still, one may expect that anomalies and 2- and 3-point correlators of currents of the (2,0) theory may have special "protected" form, with simple dependence on N, allowing one to interpolate between $N \gg 1$ and $N = 1$ cases.

This was, in fact, observed for the R-symmetry anomaly of the (2,0) theory [11]: the anomaly of the (2,0) theory obtained from the 11-d action containing the standard supergravity term plus a higher-derivative $\mathcal{C}_3 R^4$ term [12] is given by the sum of the leading supergravity $O(N^3)$ and subleading $O(N)$ terms, and for $N = 1$ is equal to the R-symmetry anomaly corresponding to the single tensor multiplet [13,14].

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[1] The same is true also for the correlators of R-symmetry currents [7].

Since the conformal and R-symmetry anomalies of the (2,0) theory should belong to the same $d = 6$ supermultiplet [15,11], one should then expect to find a similar $O(N)$ correction to the $O(N^3)$ supergravity contribution [8] to the (2,0) conformal anomaly. This $O(N)$ correction should originate from a higher-derivative R^4 term in the 11-d action which should be a part of the same superinvariant as $\mathcal{C}_3 R^4$ term (just like the second-derivative supergravity terms R and $\mathcal{C}_3 F_4 F_4$ are).

Our aim below is to discuss a mechanism of how this may happen. We shall argue that the 11-d action contains a particular R^4 term, which, upon compactification on S^4, leads to a special combination of R^3 terms in the effective 7-d action. These R^3 corrections produce extra $O(N)$ terms in the conformal anomaly of the boundary (2,0) conformal theory. As a result, the coefficient $4N^3$ in the ratio of the (2,0) theory and tensor multiplet scale anomalies may be shifted to $4N^3 - 3N$. Since the latter is equal to 1 for $N = 1$, this would be a resolution of the "$4N^3$" puzzle.

Since this conclusion is sensitive to numerical values of coefficients in the 11-d low energy effective action we shall start with a critical review of what is known about the structure of R^4 terms in type IIA string theory in 10-d and their counterparts in M-theory. While the type IIB theory effective action contains the same $J_0 \sim R^4$ invariant at the tree and one-loop levels, the one-loop term in type IIA theory is a combination of two different R^4 structures. We shall argue that they should be organized into two different $\mathcal{N} = 2A$ superinvariants – J_0 and $\mathcal{I}_2$ (containing P-odd $B_2 \mathrm{tr} R^4$ term) in a way different than it was previously suggested (Section 2). The corresponding tow $D = 10$ superinvariants "lifted" to $D = 11$ represent the leading R^4 corrections to the 11-d supergravity action (Section 3).

These terms should be supplemented by proper $F_4 = d\mathcal{C}_3$ dependent terms as required by supersymmetry and chosen in a specific "on-shell" scheme not to modify the $AdS_7 \times S^4$ solution of the $D = 11$ supergravity. Assuming that, in Section 4 we discuss higher derivative corrections to the 7-d action of S^4 compactified theory which follow from the presence of the R^4 terms in $D = 11$ action. In Section 5 we compute the corresponding $O(N)$ contributions to the scale anomaly of the (2,0) theory using the method of [8], and draw analogy between the total $O(N^3) + O(N)$ result and the expression for the R-symmetry anomaly found in [11].

2. R^4 terms in 10 dimensions

Let us start with a review of the structure of the R^4 terms in the effective actions of type IIA superstring in 10 dimensions and the corresponding terms in M-theory effective action in 11 dimensions, paying special attention to explicit values of numerical coefficients.

The relevant terms in the tree + one loop type IIA string theory effective action can be written in the form

$$S = S_0 + S_1 \ ,$$

$$S_0 = \frac{1}{2\kappa_{10}^2} \int d^{10}x \sqrt{-G}\ e^{-2\phi} \ \left(R - \frac{1}{2 \cdot 3!} H_3^2 + ... + b_0 \alpha'^3 J_0 \right) \ , \tag{2.1}$$

$$S_1 = \frac{1}{2\pi\alpha'} \int d^{10}x \sqrt{-G}\ \ L_1 \ , \qquad\qquad L_1 = b_1 \mathcal{J}_0 + b_2 K \ , \tag{2.2}$$

where $H_{mnk} = 3\partial_{[m} B_{nk]}$ and[2]

$$J_0 = \mathcal{J}_1 + \mathcal{J}_2 \equiv t_8 \cdot t_8 RRRR + \frac{1}{4 \cdot 2!} \epsilon_{10} \cdot \epsilon_{10} RRRR \ , \tag{2.3}$$

$$\mathcal{J}_0 = \mathcal{J}_1 - \mathcal{J}_2 \equiv t_8 \cdot t_8 RRRR - \frac{1}{4 \cdot 2!} \epsilon_{10} \cdot \epsilon_{10} RRRR \ , \tag{2.4}$$

$$K = \epsilon_{10} B_2 [\mathrm{tr} R^4 - \frac{1}{4} (\mathrm{tr} R^2)^2] \ . \tag{2.5}$$

In the notation we are using the numerical coefficients are

$$2\kappa_{10}^2 = (2\pi)^7 g_s^2 \alpha'^4 \ , \tag{2.6}$$

$$b_0 = \frac{1}{3 \cdot 2^{11}} \zeta(3) \ , \qquad b_1 = \frac{1}{(2\pi)^4 \cdot 3^2 \cdot 2^{13}} \ , \qquad b_2 = -12 b_1 = -\frac{1}{(2\pi)^4 \cdot 3 \cdot 2^{11}} \ . \tag{2.7}$$

The tree and one-loop coefficients of the well-known $\mathcal{J}_1 = t_8 \cdot t_8 RRRR$ term[3] can be determined from the 4-graviton amplitude [17,18,19].[4]

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[2] We use Minkowski notation for the metric and ϵ tensor, so that $\epsilon_{10}\epsilon_{10} = -10!$, and upon reduction to 8 spatial dimensions $\epsilon_{mn...}\epsilon_{mn...} \to -2\epsilon_8\epsilon_8$. For other notation see also [16].

[3] The more explicit form of this term is $\mathcal{J}_1 = 24 t_8 [\mathrm{tr} R^4 - \frac{1}{4}(\mathrm{tr} R^2)^2]$, where $R = (R^{ab}{}_{mn})$ and $t_8 \mathrm{tr} R^4 \equiv \mathrm{tr}\big(16 R_{mn} R_{rn} R_{ml} R_{rl} + 8 R_{mn} R_{rn} R_{rl} R_{ml} - 4 R_{mn} R_{mn} R_{rl} R_{rl} - 2 R_{mn} R_{rl} R_{mn} R_{rl}\big)$.

[4] Note that the total coefficient of the $t_8 \cdot t_8 RRRR$ term in S is thus $-\frac{1}{(2\pi)^7 \cdot 3 \cdot 2^{11} \alpha'} (\frac{\zeta(3)}{g_s^2} + \frac{\pi^2}{3})$. The relative combination $\frac{\zeta(3)}{g_s^2} + \frac{\pi^2}{3}$ is the same as in [19] (where $g^2 = (2\kappa_{10})^2 (2\alpha')^{-4} = 16\pi^7 g_s^2$) and in [20], but our overall normalization of this term is different (by factor 2^5 compared to [20]).

The invariant $\mathcal{J}_2 = \frac{1}{4\cdot 2!}\epsilon_{10}\cdot\epsilon_{10}RRRR$ which will play important role in what follows is the $D=10$ extension of the integrand of the Euler invariant in 8 dimensions

$$\mathcal{J}_2 = \frac{1}{4}E_8 \ , \qquad E_8 = \frac{1}{(D-8)!}\epsilon_D\epsilon_D R^4 = \pm 8!\delta^{n_1}_{[m_1}\dots\delta^{n_8}_{m_8]}R^{m_1m_2}{}_{n_1n_2}\dots R^{m_7m_8}{}_{n_7n_8} \ , \quad (2.8)$$

where $\pm$ correspond to the case of Euclidean or Minkowski signature.[5]

The expansion of E_8 near flat space ($g_{mn} = \eta_{mn} + h_{mn}$) starts with h^5 terms (see, e.g., [21]), so that its coefficient cannot be directly determined from the on-shell 4-graviton amplitude. The sigma-model approach implies [22,23] that E_8 does appear in S_0, i.e. that (up to usual field redefinition ambiguities) the tree-level type II string R^4 term is indeed proportional to J_0 (2.3).

The structure of the kinematic factor $(t_8 + \frac{1}{2}\epsilon_8)(t_8 + \frac{1}{2}\epsilon_8)$ in the one-loop type IIA 4-point amplitude with transverse polarisations and momenta suggests [24,25,26] that the one-loop R^4 terms in $D=10$ type IIA theory should be proportional to the opposite-sign combination $\mathcal{J}_0$ (2.4) of the $\mathcal{J}_1$ and $\mathcal{J}_2$ terms, and this assumption passes some compactification tests [25,26].

The presence of the P-odd one-loop term K (2.5) can be established [27] following similar calculations of anomaly-related terms in the heterotic string [28]. Its coefficient b_2 can be fixed by considering compactification to 2 dimensions [27], and its value is in agreement with the coefficient required by 5-brane anomaly cancellation [12] (see also below).

The low-energy effective string action should be supersymmetric.[6] Remarkably, the coefficients in (2.7) are indeed consistent with what is known about the structure of possible R^4 super-invariants. First, the h^4 term in $t_8t_8R^4$ is the bosonic part of the on-shell linearized superspace invariant [30] (i.e. $\int d^{16}\theta\ \Phi^4, \quad \Phi = \phi + .. + \theta^4 R + ...$ written in terms of $\mathcal{N}=1$ or $\mathcal{N}=2B$ [31,32] on-shell superspace superfield Φ). If one first restricts consideration to $\mathcal{N}=1, \ D=10$ supersymmetry only, then one can use the classification

[5] The Euler number in 8 dimensions is $\chi = \frac{1}{(4\pi)^4\cdot 3\cdot 2^7}\int d^8x\sqrt{g}\ E_8$.

[6] The string S-matrix is invariant under on-shell supersymmetry, so the leading-order corrections to effective action evaluated on the supergravity equations of motion should be invariant under the standard supersymmetry transformations. Since the $D=10$ supersymmetry algebra does not close off shell, the full off-shell effective action should be invariant under "deformed" supersymmetry transformations (see, e.g., [29]).

of possible bosonic R^4 parts of on-shell non-linear $\mathcal{N}=1$ superinvariants given in [33]. A basis of the three independent $\mathcal{N}=1$ invariants [33,16] can be chosen as J_0, X_1, X_2

$$J_0 = t_8 \cdot t_8 RRRR + \frac{1}{4 \cdot 2!} \epsilon_{10} \cdot \epsilon_{10} RRRR \ , \tag{2.9}$$

$$X_1 = t_8 \mathrm{tr} R^4 - \frac{1}{4} \epsilon_{10} B_2 \mathrm{tr} R^4 \ , \qquad X_2 = t_8 \mathrm{tr} R^2 \mathrm{tr} R^2 - \frac{1}{4} \epsilon_{10} B_2 \mathrm{tr} R^2 \mathrm{tr} R^2 \ . \tag{2.10}$$

One may try to combine these $\mathcal{N}=1$ invariants to form potential $\mathcal{N}=2A$ superinvariants. Since $t_8 t_8 R^4 = 24 t_8 [\mathrm{tr} R^4 - \frac{1}{4} \mathrm{tr} R^2 \mathrm{tr} R^2]$, one may consider two candidate invariants which contain combinations of $\mathcal{J}_1$ (2.3) or $\mathcal{J}_2$ (2.4) with $\pm 6K$ (2.5), i.e.

$$\mathcal{I}_1 = 24(X_1 - \frac{1}{4} X_2) = \mathcal{J}_1 - 6K$$

$$= \ t_8 \cdot t_8 RRRR - 6 \epsilon_{10} B_2 [\mathrm{tr} R^4 - \frac{1}{4} (\mathrm{tr} R^2)^2] \ , \tag{2.11}$$

or

$$\mathcal{I}_2 = J_0 - 24(X_1 - \frac{1}{4} X_2) = \mathcal{J}_2 + 6K$$

$$= \frac{1}{4 \cdot 2!} \epsilon_{10} \cdot \epsilon_{10} RRRR + 6 \epsilon_{10} B_2 [\mathrm{tr} R^4 - \frac{1}{4} (\mathrm{tr} R^2)^2] \ , \tag{2.12}$$

$$\mathcal{I}_1 + \mathcal{I}_2 = J_0 \ . \tag{2.13}$$

The 1-loop term L_1 (2.2) with $b_2 = -12 b_1$ can thus be represented as a combination of *two* different R^4 superinvariants [24,25], i.e. as

$$L_1 = b_1 \mathcal{J}_0 + b_2 K = b_1 (\mathcal{J}_1 - \mathcal{J}_2 - 12K) = b_1 (-J_0 + 2\mathcal{I}_1) \ , \tag{2.14}$$

or as

$$L_1 = b_1 (J_0 - 2\mathcal{I}_2) \ . \tag{2.15}$$

The J_0-term should represent a separate $\mathcal{N}=2$ invariant.[7] A non-trivial question is which of $\mathcal{I}_1$ and $\mathcal{I}_2$ can be actually extended to an invariant of $\mathcal{N}=2A$ supersymmetry.[8]

[7] In [33] where non-linear extensions of $\mathcal{N}=1$ on-shell R^4 superinvariants were constructed the transformation of the dilaton prefactor was ignored. As a result, one was not able to make a distinction between J_0 terms appearing at the tree and 1-loop levels. It is natural to conjecture that $f(\phi) J_0$ terms should combine into an $\mathcal{N}=2A$ superinvariant (invariant under deformed supersymmetry). For a discussion of supersymmetry of $e^{-2\phi} R + f(\phi) J_0$ action in type IIB supergravity theory see [34].

[8] Once the dilaton dependence of J_0 terms is taken into account, one will not be able to freely switch between $\mathcal{I}_1$ and $\mathcal{I}_2$ using (2.13).

We would like to argue that it is $\mathcal{I}_2$ and not $\mathcal{I}_1$ that is the true $\mathcal{N} = 2A$ superinvariant. Namely, it is the Euler term $\mathcal{J}_2 = \frac{1}{4}E_8$ and *not* $\mathcal{J}_1 = t_8 t_8 RRRR$ that is the "superpartner" of the B_2-dependent term K (2.5). The form of the 1-loop correction L_1 that admits a super-extension is then (2.15) and not (2.14). Then the tree + one-loop J_0 terms in the type IIA theory will be exactly the *same* as in the type IIB theory, $-\frac{1}{(2\pi)^4 \cdot 3 \cdot 2^{13} \alpha'}(\frac{\zeta(3)}{g_s^2} + \frac{\pi^2}{3})J_0$, with the type IIA theory action containing in addition one extra one-loop contribution (2.15) proportional to the superinvariant $\mathcal{I}_2$.

Indeed, the weak-field expansions of both E_8 and K start with 5-order terms, and the corresponding 5-point amplitudes should be related by global supersymmetry. At the same time, it is hard to imagine how the linearized on-shell "$\mathcal{W}^4$" $\mathcal{N} = 2$ superspace invariant corresponding to h^4 term in $t_8 t_8 RRRR$ may have a non-linear extension containing P-odd term K.

A more serious argument against $t_8 t_8 RRRR$ being a "superpartner" of $\epsilon_{10} B_2 [\mathrm{tr} R^4 - \frac{1}{4}(\mathrm{tr} R^2)^2]$ is the following. The $D = 10$ type II supergravity is known to contain a one-loop quadratic Λ^2 UV divergence proportional to $t_8 t_8 RRRR$ (this can be seen [35] by taking the field theory limit, $\alpha' \to 0$, $\Lambda =$fixed, in the one-loop 4-graviton amplitude, cf. (2.2)). At the same time, the Chern-Simons type terms like $\epsilon_{10} B_2 R^4$ can not appear in the *UV divergent* part of one-loop effective action.[9] This can be proved directly by using the background field method: all one-loop UV divergent terms must be manifestly invariant under 2-form gauge transformations and as well as diffeomorphisms. Since, e.g., a proper time cutoff is expected to preserve supersymmetry at the level of one-loop UV divergences, one concludes that $\mathcal{J}_1$ and K can not be parts of the same superinvariant.

Similar argument can be given in the context of $D = 11$ theory. The $t_8 t_8 RRRR$ term appears [36,20,37,24] as a cubic UV divergence (with a particular value of the UV cutoff being fixed by duality considerations [37]), but $\epsilon_{11} \mathcal{C}_3 R^4$ term [12] can have only a finite coefficient (with a non-perturbative dependence on κ_{11} on dimensional grounds). Thus (contrary to some previous suggestions in the literature, cf. [20,24,25,38]) these terms can not be related by supersymmetry, and the superpartner of the $\epsilon_{11} \mathcal{C}_3 R^4$ term should be the $D = 11$ analog of $\mathcal{J}_2 = \frac{1}{4}E_8$ (see section 3).

Before turning to a detailed discussion of the $D = 11$ terms, let us add few comments about the structure of the $D = 10$ effective action (2.1),(2.2). In addition to the R^4 terms

[9] Known examples of induced CS terms have finite coefficients and originate from IR effects (they appear from 1-loop contributions containing $\frac{1}{\partial^2}$ massless poles, and thus can be re-written in a manifestly gauge invariant but nonlocal form).

given explicitly in (2.3) and (2.4), it may contain also other Ricci tensor dependent terms as well as terms depending on other fields (cf. [39]), for example, terms involving two and more powers of $H_3 = dB_2$ (which were not included in the discussion of super-invariants in [33]). The well-known field redefinition ambiguity [18,40] allows one to change the coefficients of "on-shell" terms.[10] In particular, the tree-level effective action (2.1) may contain other R_{mn} dependent terms in addition to the full curvature contractions present in J_0 (see [23,41,33])

$$J_0 = 3 \cdot 2^8 (R^{hmnk} R_{pmnq} R_h{}^{rsp} R^q{}_{rsk} \ + \ \tfrac{1}{2} R^{hkmn} R_{pqmn} R_h{}^{rsp} R^q{}_{rsk}) + O(R_{mn}) \ . \quad (2.16)$$

The field redefinition ambiguity allows one to choose the action in a specific "scheme" where only the Weyl tensor part of the curvature appears in J_0, i.e.

$$J_0 \ \to \ \hat{J}_0 \ = 3 \cdot 2^8 (C^{hmnk} C_{pmnq} C_h{}^{rsp} C^q{}_{rsk} \ + \ \tfrac{1}{2} C^{hkmn} C_{pqmn} C_h{}^{rsp} C^q{}_{rsk}) \ . \qquad (2.17)$$

That freedom of choice of a special scheme is crucial, in particular, in order to avoid corrections to certain highly symmetric leading-order solutions, both in 10 and in 11 dimensions (see section 3). For example, in type IIB theory the (scale of) $AdS_5 \times S^5$ solution is not modified by the R^4 terms [42] only in the scheme [43] where they have the form (2.17).

3. R^4 terms in 11 dimensions

Since the invariant $\mathcal{I}_2$ in (2.15) contains the P-odd CS type part K, its coefficient can not develop dilaton dependence without breaking B_2 gauge invariance, i.e. its value can not be renormalized from its coupling-independent one-loop value [16]. Taking the limit $g_s \to \infty$ this term can then be lifted to a corresponding superinvariant in $D = 11$ theory. Assuming that the coefficient of the J_0 invariant (2.3) does not receive higher than one loop perturbative string corrections, it can be also lifted [20,24,25,26] to $D = 11$ (with its tree-level part giving vanishing contribution). The resulting presence of the $t_8 t_8 R^4$ term in the M-theory effective action is indeed in agreement with what follows directly from the low-energy expansion of the 4-graviton amplitude in $D = 11$ supergravity [37,24].

In view of the above discussion, we conclude that the effective action of the $D = 11$ theory should contain two distinct R^4 superinvariants: (i) J_0 with $t_8 t_8 R^4$ as its part,

[10] For example, ignoring other fields, one may use $R_{mn} = 0$ to simplify the structure of R^4 invariants as the graviton legs in the string amplitudes they correspond to are on mass shell.

and (ii) $\mathcal{I}_2$ which is a sum of the E_8 and $\epsilon_{11}\mathcal{C}_3 R^4$ structures. With this separation, the coefficient in front of the J_0 term is then in agreement with the 4-graviton amplitude (with the M-theory cutoff [37]), and the coefficient of the $\mathcal{I}_2$ term (its $\mathcal{C}_3 R^4$ part) is precisely the one implied by the M5 brane anomaly cancellation condition [12].

Explicitly, the $D = 11$ action is then (cf. (2.1),(2.2))

$$\mathcal{S} = \mathcal{S}_0 + \mathcal{S}_1 \ ,$$

$$\mathcal{S}_0 = \frac{1}{2\kappa_{11}^2} \int d^{11}x \ \sqrt{g} \ \left[R - \frac{1}{2 \cdot 4!} F_4^2 - \frac{1}{6 \cdot 3! \cdot (4!)^2} \epsilon_{11} \mathcal{C}_3 F_4 F_4 \right] \ , \tag{3.1}$$

$$\mathcal{S}_1 = b_1 T_2 \int d^{11}x \sqrt{g} \ \left(J_0 - 2\mathcal{I}_2 \right) \ . \tag{3.2}$$

Here $F_{mnkl} = 4\partial_{[m}\mathcal{C}_{nkl]}$ and the two R^4 super-invariants are (see (2.9),(2.8),(2.11))

$$J_0 = t_8 \cdot t_8 RRRR + \frac{1}{4} E_8 \ , \qquad E_8 = \frac{1}{3!} \epsilon_{11} \cdot \epsilon_{11} RRRR \ , \tag{3.3}$$

$$\mathcal{I}_2 = \frac{1}{4} E_8 + 2\epsilon_{11}\mathcal{C}_3 [\mathrm{tr} R^4 - \frac{1}{4} (\mathrm{tr} R^2)^2] \ . \tag{3.4}$$

The constant $b_1 = \frac{1}{(2\pi)^4 \cdot 3^2 \cdot 2^{13}}$ is the same as in (2.7) and the 10-d and 11-d parameters are related as follows (T_1 and T_2 are the string and the membrane tensions)[11]

$$2\kappa_{11}^2 = (2\pi)^5 l_{11}^9 \ , \qquad \kappa_{10}^2 = \frac{\kappa_{11}^2}{2\pi R_{11}} \ , \qquad l_{11} = (2\pi g_s)^{1/3} \sqrt{\alpha'} \ , \qquad R_{11} = g_s \sqrt{\alpha'} \ , \tag{3.5}$$

$$T_2 = \frac{1}{2\pi l_{11}^3} = (2\pi)^{2/3} (2\kappa_{11}^2)^{-1/3} \ , \qquad T_1 = \frac{1}{2\pi\alpha'} = 2\pi R_{11} T_2 \ . \tag{3.6}$$

The subleading $O(T_2)$ term (3.2) in the effective action of 11-d theory may contain also other $O(R_{mn})$ and $O(F_4)$ terms. The invariant J_0 (supplemented with appropriate F_4 dependent terms) may be considered as a non-linear extension of the linearized "R^4" superinvariant in on-shell $D = 11$ superspace [44]. The P-even part of the second super-invariant starting with $\mathcal{I}_2$ (3.4) may also include extra $O(F_4)$ terms. Note that in the exterior form notation $\mathcal{I}_2$ may be written as

$$\mathcal{I}_2 \ e^0 \wedge e^1 \wedge ... \wedge e^{10} \ = \ \frac{2}{3} \epsilon_{11} e \wedge e \wedge e \wedge R \wedge R \wedge R \wedge R$$

$$+ \ 2^5 \cdot 3! \ \mathcal{C}_3 \wedge \left[\mathrm{tr}(R \wedge R \wedge R \wedge R) - \frac{1}{4} \mathrm{tr}(R \wedge R) \wedge \mathrm{tr}(R \wedge R) \right] \ . \tag{3.7}$$

[11] Note that B_2 and $\mathcal{C}_3$ are canonically normalized, so that the 10-d invariant $T_1 \int B_2 \wedge \mathrm{tr}(\wedge R)^4$ in (2.2) goes into the 11-d one $T_2 \int \mathcal{C}_3 \wedge \mathrm{tr}(\wedge R)^4$, where in the form notation $B_2 = \frac{1}{2} B_{mn} dx^m \wedge dx^n$, $\mathcal{C}_3 = \frac{1}{3!} \mathcal{C}_{mnk} dx^m \wedge dx^n \wedge dx^k$, $R^{ab} = \frac{1}{2} R^{ab}_{mn} dx^m \wedge dx^n$. Thus $\mathcal{S}_1$ (3.2) contains $T_2 \int \mathcal{C}_3 \wedge \mathrm{tr}(\wedge R)^4$ with the coefficient $4 \cdot 3! \cdot 2^4 b_1 = \frac{1}{(2\pi)^4 \cdot 3 \cdot 2^6}$ which is the same as in [12].

4. $AdS_7 \times S^4$ solution and compactification on S^4

The $D = 11$ supergravity admits the well known $AdS_7 \times S^4$ solution with F_4 flux N through S^4 [45]. Compactifying on S^4, one may derive the corresponding $d = 7$ supergravity action, which gives the $O(N^3)$ contribution [8] to the conformal anomaly in the corresponding boundary conformal (2,0) theory.

Let us consider how the presence of the R^4 terms in the 11-d effective action $\mathcal{S}_1$ (3.2) may influence the existence of the $AdS_7 \times S^4$ solution and expansion near it. Using the on-shell superspace description of 11-d supergravity and assuming that all local higher-order corrections to the equations of motion can be written again in terms of the basic on-shell supergravity superfield, it was argued in [46] that these corrections cannot modify the maximally supersymmetric $AdS_7 \times S^4$ solution. It should be possible to see explicitly that adding the J_0 term in (3.2) (supplemented with F_4 dependent terms as required by supersymmetry[12] and chosen in a special "on-shell" scheme analogous but not equivalent[13] to (2.17) in 10-d theory) does not change the leading-order $AdS_7 \times S^4$ solution. One may view J_0 as originating from a restricted superspace integral of $\mathcal{W}^4$, where $\mathcal{W}_{abcd}(x, \theta)$ is the on-shell supergravity superfield [44], which has the structure $\mathcal{W} = F_4 + ... + \theta\theta(\gamma...\gamma R + \gamma...\gamma F_4 F_4 + \gamma...\gamma D F_4) + ...$ ($\gamma...\gamma$ stand for products of gamma matrices). Then $J_0 \sim (R + F_4 F_4)^4$ and its first, second and third variation over the metric evaluated on $AdS_7 \times S^4$ + F_4-flux background ($R_{mn} \sim (F_4^2)_{mn},\ \partial F_4 = 0$) will vanish, essentially as in the case of $AdS_5 \times S^5$ solution of type IIB theory corrected by J_0 term [42] (taken in the form (2.17)).[14]

The fact that the $AdS_7 \times S^4$ solution (and, in particular, the radii of its factors) is not modified by the J_0 correction can be also represented as a consequence of the fact that upon compactification of the 11-d theory on S^4 with F_4 flux the J_0 term (taken in

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[12] In addition to F_4 dependent terms (which may contain up to 8 powers of F_4) there are also ∂F_4 dependent terms which accompany $t_8 t_8 R^4$ part of J_0 in the 4-point S-matrix [47] (as suggested by the analysis of tree-level 4-point scattering amplitudes in 11-d supergravity). These derivative terms vanish on $AdS_7 \times S^4$ background.

[13] Note that in contrast to $AdS_5 \times S^5$ space with equal radii the 11-d space $AdS_7 \times S^4$ space with radii 1 and $\frac{1}{2}$ is not conformally flat.

[14] The vanishing of the first variation is equivalent to the vanishing of the first correction to the 11-d supergravity equations of motion $\gamma^{abc} D\mathcal{W}_{abcd} = 0$ due to the supercovariant constancy of $\mathcal{W}$ [46]. The argument of [46] should certainly apply to the first subleading correction to the 11-d supergravity equations of motion coming from R^4 terms in the action.

the special "on-shell" scheme) reduces to the Weyl tensor dependent C^4 term (2.17), now defined in 7 dimensions.[15] This term produces an $O(N)$ correction [43] to the leading N^3 term [48] in the entropy of (2,0) theory describing multiple M5 branes. As in the $AdS_5 \times S^5$ case in type IIB theory, this C^4 term does not, however, modify the expression for the conformal anomaly of the boundary conformal theory.[16]

Let us now discuss the second invariant $\mathcal{I}_2$ (3.4) in (3.2). It is easy to see that its P-odd part $\epsilon_{11}\mathcal{C}_3[\mathrm{tr}R^4 - \frac{1}{4}(\mathrm{tr}R^2)^2]$ does not modify the $AdS_7 \times S^4$ solution. Upon reduction on S^4 it leads to $O(N)$ CS terms in $d=7$ action [11]. As for the E_8 part of $\mathcal{I}_2$, we shall assume that, as in the case of J_0, there exists an "on-shell" scheme in which this term, supplemented with proper F_4-dependent terms, also does not modify the leading-order $AdS_7 \times S^4$ solution.

The main point is that upon compactification on S^4 the E_8 term in (3.4) should produce additional R^3 higher-derivative terms in the 7-d effective action which, while not changing the vacuum solution, will give subleading $O(N)$ corrections to the conformal anomaly of the boundary CFT.[17]

It is known that the $\mathcal{C}_3 R^4$ part of $\mathcal{I}_2$ (3.4) gives a subleading $O(N)$ correction to the R-symmetry anomaly of the (2,0) theory [12,11]. Since the R-symmetry and conformal anomalies should belong to the same 6-d supermultiplet, it is natural to expect that the "superpartner" of the $\mathcal{C}_3 R^4$ term, i.e. the E_8 term in $\mathcal{I}_2$, should lead to an $O(N)$ correction to the conformal anomaly of the boundary 6-d theory. This is what we are going to suggest below.

Since we do not know the F_4 (and R_{mn}) dependent terms which supplement E_8 to a superinvariant, to determine the terms in the 7-d action that originate from the E_8 part of the invariant $\mathcal{I}_2$ in (3.2) we shall use the following heuristic strategy. We shall start with

[15] The tree + one-loop J_0 term in type IIB theory leads to the same C^4 term (2.17) in the 5-d effective action obtained by compactifying the type IIB theory on S^5 with F_5 flux.

[16] It is important to stress for what follows that in the above discussion we treated J_0 (3.3) as a whole, without splitting it into $t_8 t_8 R^4$ and E_8 parts. It is only that particular combination of R^4 terms that takes the "irreducible" form (2.16) (cf. [41]), and thus should lead only to C^4 terms upon compactification to $d=7$. At the same time, E_8 contains "reducible" curvature contractions like $((R_{mnkl})^2)^2 + R(R_{mnkl})^3 + ...$ and thus may, in principle, lead to $O(R^n)$, $n<4$, terms upon compactification to $d=7$.

[17] These terms will give also another $O(N)$ correction to the entropy of (2,0) theory, in addition to the one coming from the J_0 term (2.17) found in [43].

E_8 and compute it in the case when the 11-d space is a direct product, $M^{11} = M^7 \times M^4$. It is easy to see that

$$E_8(M^7 \times M^4) = 4E_2(M^4)E_6(M^7) + 12E_4(M^4)E_4(M^7) \ , \tag{4.1}$$

where, as in (2.8),

$$E_{2n}(M^d) \equiv \frac{1}{(d-2n)!}\epsilon_d \cdot \epsilon_d R^n \ , \qquad d \geq 2n \ , \tag{4.2}$$

and $E_{2n}(M^d) = 0$ for $d < 2n$. In the case when M^4 is a 4-sphere of radius L ($R_{S^4} = \frac{12}{L^2}$) and M^7 has curvature R we get

$$\begin{aligned} E_8(M^7 \times S^4) &= \frac{3 \cdot 2^5}{L^2} E_6(M^7) + \frac{3^2 \cdot 2^7}{L^4} E_4(M^7) \\ &= \frac{3 \cdot 2^5}{L^2} \epsilon_7 \epsilon_7 RRR + \frac{3 \cdot 2^6}{L^4} \epsilon_7 \epsilon_7 RR \ . \end{aligned} \tag{4.3}$$

A remarkable property of the E_8 invariant is that it does not produce a correction to the cosmological or Einstein term in the 7-d action.

Next, we shall assume that when the same reduction is repeated for the analog of E_8 term in a special "on-shell" scheme (i.e. for E_8 supplemented by F_4 and R_{mn} dependent terms so that it does not produce a modification of the leading-order $AdS_7 \times S^4$ solution) then the resulting terms in the 7-d action will be the same as in (4.3) but with the curvature tensor R of M^7 replaced by its Weyl tensor C part.

In what follows we shall consider only on the $E_6(M^7) \sim C^3 + ...$ term in (4.3) coming from E_8. The reason is that we shall compute the corresponding contribution to the scale anomaly of the boundary theory only modulo R_{mn}-dependent terms, but it is easy to see that a potential C^2 term in the 7-d action (coming from E_4 in (4.3)) can lead only to terms in the conformal anomaly which vanish when the 6-d boundary space is Ricci flat.

Choosing the normalization in which the radii of AdS_7 and S^4 are 1 and $L = \frac{1}{2}$ so that $\text{Vol}(S^4) = \frac{8\pi^2}{3} L^4 = \frac{\pi^2}{6}$, and assuming that the value of the quantized F_4 flux is N, we get (see (3.5),(3.6) and [48,43])

$$\frac{1}{2\kappa_{11}^2} = \frac{N^3}{2^8 \pi^5 L^9} = \frac{2N^3}{\pi^5} \ , \qquad \frac{1}{2\kappa_7^2} = \frac{\text{Vol}(S^4)}{2\kappa_{11}^2} = \frac{N^3}{3\pi^3} \ , \qquad T_2 = \frac{2N}{\pi} \ . \tag{4.4}$$

The relevant $-\int[N^3(R-2\lambda)+NC^3]$ terms in the 7-d action[18] are then

$$S^{(7)} = -\frac{N^3}{3\pi^3}\int d^7x\ \sqrt{g}\ \ (R+30) + \frac{\gamma N}{3^2\cdot 2^{11}\cdot\pi^3}\int d^7x\ \sqrt{g}\ \hat{E}_6 + ... \ , \tag{4.5}$$

where the explicit form of the $\hat{E}_6 \sim C^3$ correction term is (cf. (2.8))

$$\hat{E}_6 = (E_6)_{R_{mn}=0} \ = \ \epsilon_7\epsilon_7 CCC$$

$$= \ -6!\delta^{n_1}_{[m_1}\ldots\delta^{n_6}_{m_6]}C^{m_1m_2}{}_{n_1n_2}C^{m_3m_4}{}_{n_3n_4}C^{m_5m_6}{}_{n_5n_6} \ = \ -\ 32\ (2I_1+I_2)\ , \tag{4.6}$$

where I_1 and I_2 are defined as

$$I_1 = C_{amnb}C^{mpqn}C_p{}^{ab}{}_q\ , \qquad\qquad I_2 = C_{ab}{}^{mn}C_{mn}{}^{pq}C_{pq}{}^{ab}\ . \tag{4.7}$$

As follows from (4.3) the numerical coefficient γ is

$$\gamma = 1\ , \tag{4.8}$$

but we shall keep it arbitrary, given the uncertainties in the above derivation of the correction term in (4.5) (for example, the presence of $(F_4)^2(R_{mnkl})^3$ terms in $\mathcal{I}_2$ would shift the value of γ).

5. Conformal anomaly of (2,0) theory

Let us now determine the contribution of the C^3 correction term in the 7-d action (4.5) which originated from the E_8 part of the $\mathcal{I}_2$ superinvariant in the 11-d action (3.2) to the conformal anomaly of the $d=6$ boundary conformal theory. We shall follow the same method as used in [8] in computing the leading N^3 term in the anomaly.[19] We shall compute only the $O(N)$ contribution to the scale anomaly (which is the same as integrated conformal anomaly, assuming topology of 6-space is trivial) and ignore terms which depend on R_{mn}, i.e. concentrate only on the Weyl-invariant non total derivative C^3 terms ("type B" part) in the 6-d conformal anomaly.

[18] Here we consider the Euclidean signature and change overall sign of the action, i.e. $\int R \to -\int R$.

[19] Similar computation of subleading corrections to conformal anomaly of 4-d boundary conformal field theories (with $\mathcal{N} < 4$ supersymmetry) coming from R^2 curvature terms in 5-d effective action were discussed in [49,50,51,38].

To obtain the conformal anomaly one is to solve the 7-d equations for the metric (as in (4.4) we set the radius of AdS_7 to be equal to 1)

$$ds^2 = \frac{1}{4}\rho^{-2}d\rho^2 + \rho^{-1}g_{ij}(x,\rho)dx^i dx^j \ , \tag{5.1}$$

evaluate the action on the solution $g = g_0(x) + \rho g_2(x) + ...$, and compute its variation under the Weyl rescaling of the 6-d boundary metric. The anomaly is essentially determined by the coefficient of the logarithmic divergence produced by the integral over ρ [8]. In the present case of (4.5) we find (using (4.5),(4.6) and $R_{AdS_7} = -42$)

$$S^{(7)} = \int d^6x \Big[\frac{N^3}{3\pi^3} \cdot 6 \cdot \int_\epsilon \frac{d\rho}{\rho^4} \sqrt{g(x,\rho)} \ - \ \frac{\gamma N}{3^2 \cdot 2^6 \cdot \pi^3} \cdot \frac{1}{2} \cdot \int_\epsilon \frac{d\rho}{\rho} \sqrt{g}\ (2I_1 + I_2) + ... \Big] \ . \tag{5.2}$$

Since [8]

$$6 \int_\epsilon \frac{d\rho}{\rho^4} \sqrt{g(x,\rho)} \ = \ \sqrt{g_0}\ \big[a_0(x)\epsilon^{-3} + ... - a_6(x) \ln \epsilon \big] + ... \ , \tag{5.3}$$

the anomaly is given by the sum of the $O(N^3)$ and $O(N)$ terms[20]

$$\mathcal{A}_{(2,0)} = \mathcal{A}^{N^3}_{(2,0)} + \mathcal{A}^{N}_{(2,0)} = -\frac{N^3}{3\pi^3} \cdot 2a_6 \ + \ \frac{\gamma N}{3^2 \cdot 2^6 \cdot \pi^3}\ (2I_1 + I_2 + ...) \ . \tag{5.4}$$

Here a_6 and I_1, I_2 are evaluated for the boundary metric g_0, and dots stand for $O(N)$ R_{mn}-dependent and total derivative terms we are ignoring.

The result of [8] for the leading-order contribution $\mathcal{A}^{N^3}_{(2,0)}$ written as a sum of the type A (Euler), type B (Weyl invariant) and scheme-dependent (covariant total derivative) terms [52,53] is

$$\mathcal{A}^{N^3}_{(2,0)} = -\frac{4N^3}{(4\pi)^3 \cdot 3^2 \cdot 2^5} \Big[E_6 + 8(12I_1 + 3I_2 - I_3) + O(\nabla_i J^i) \Big] \ , \tag{5.5}$$

where $E_6 = \epsilon_6 \epsilon_6 RRR$. The invariants I_1, I_2 (4.7) and I_3

$$I_3 = C_{mnbc} \nabla^2 C^{mnbc} + O(R_{mn}) + O(\nabla_i J^i) \ , \tag{5.6}$$

which form the basis of 3 Weyl invariants are the same as used in [9]. They are related to the invariants used in [52,8] as follows: $E_{(6)}, I_1, I_2$ and I_3 in [8] are equal to $\frac{1}{3^3 \cdot 2^{11}} E_6, -I_1, -I_2$

[20] To obtain the $O(N)$ contribution we evaluate the C^3 term in the 7-d action on the leading-order solution for the metric (5.1) (see [49] for a similar computation in the case of the R^2_{mnkl} action in $d = 5$), separate the C^3 part depending on the 6-d metric g_0, and omit other parts that depend on the Ricci tensor of g_0.

and $-5I_3'$, $I_3' = I_3 - \frac{8}{3}(2I_1 + I_2) - \frac{1}{12}E_6 + O(\nabla_i J^i)$, in terms of the invariants E_6, I_1, I_2 and I_3 used in [9] and here.[21]

We use this opportunity to point out that the curvature invariant $I_3 = -5I_3'$ as defined in [52,8] is *not*, in fact, covariant under Weyl transformations, contrary to what was assumed in [8] (this can be easily checked by computing it for the metric of a sphere S^6: one finds that while $I_1(S^6) = I_2(S^6) = 0, \quad I_3'(S^6) \neq 0$). The proper third Weyl invariant of type $C\nabla^2 C$ (5.6) was given in [54] and is equivalent to the Weyl invariant I_3 used in [9] and here. Since I_3 of [8] or I_3' is a mixture of the true Weyl invariants I_1, I_2, I_3 with E_6, the separation of the leading N^3 Weyl anomaly of the (2,0) theory [8] into type A and type B parts was not presented correctly in [8]. The correct separation was given in [9] and is used here.[22]

Note that modulo terms that vanish for $R_{mn} = 0$ and total derivative terms, one has the following relations (cf. (4.6))

$$E_6 = -\, 32\ (2I_1 + I_2) + O(R_{mn})\ , \qquad I_3 = 4I_1 - I_2 + O(R_{mn}) + O(\nabla_i J^i)\ , \tag{5.7}$$

so that $\mathcal{A}^{N^3}_{(2,0)}$ vanishes for $R_{mn} = 0$, as it should [8].

Eq. (5.5) is to be compared with the expression for the conformal anomaly for the free (2,0) tensor multiplet found in [9]:

$$\mathcal{A}_{tens.} = -\frac{1}{(4\pi)^3 \cdot 3^2 \cdot 2^5}\left[\frac{7}{4}E_6 + 8(12I_1 + 3I_2 - I_3) + O(\nabla_i J^i)\right]\ . \tag{5.8}$$

As was concluded in [9], the Weyl-invariant (type B) parts of the leading (2,0) theory anomaly (5.5) and the tensor multiplet anomaly (5.8) have exactly the same form, up to the overall factor $4N^3$ in (5.5).

Since we have found the $O(N)$ correction to the anomaly of the (2,0) theory in (5.4) only modulo R_{mn}-dependent and total derivative terms, we are able to compare only type B anomalies, or scale anomalies (assuming that the $d = 6$ space has trivial topology, so that we can ignore the integral of the Euler term E_6)

$$\mathbf{A}_{(2,0)} = \int d^6x\sqrt{g_0}\ \mathcal{A}_{(2,0)}\ , \qquad \mathbf{A}_{tens.} = \int d^6x\sqrt{g_0}\ \mathcal{A}_{tens.}\ .$$

[21] Our curvature tensor $R^a{}_{bmn} = \partial_m \Gamma^a_{bn} - ...$ has the opposite sign to that of [8]. Note also that [9] was assuming Euclidean signature where E_6 is defined as $-\epsilon_6\epsilon_6 RRR$.

[22] Note that when $R_{mn} = 0$ the two invariants – I_3' and I_3 – coincide, up to a covariant total derivative term. In fact, a separation of the conformal anomaly into type A and type B parts becomes ambiguous on a Ricci flat background.

Using (5.7) to express I_3 in terms of I_1 and I_2, we find from (5.5),(5.4) and (5.8)

$$\mathbf{A}^{N^3}_{(2,0)} = -\frac{4N^3}{(4\pi)^3 \cdot 3^2} \int d^6x\sqrt{g_0}\ (2I_1 + I_2)\ , \tag{5.9}$$

$$\mathbf{A}^{N}_{(2,0)} = \frac{\gamma N}{(4\pi)^3 \cdot 3^2} \int d^6x\sqrt{g_0}\ (2I_1 + I_2)\ , \tag{5.10}$$

$$\mathbf{A}_{tens.} = -\frac{1}{(4\pi)^3 \cdot 3^2} \int d^6x\sqrt{g_0}\ (2I_1 + I_2)\ . \tag{5.11}$$

The total scale anomaly of the (2,0) theory following from (4.5),(5.4) is then

$$\mathbf{A}_{(2,0)} = \mathbf{A}^{N^3}_{(2,0)} + \mathbf{A}^{N}_{(2,0)} = -\frac{4N^3 - \gamma N}{(4\pi)^3 \cdot 3^2} \int d^6x\sqrt{g_0}\ (2I_1 + I_2)\ . \tag{5.12}$$

Equivalently,

$$\mathbf{A}_{(2,0)} = -\frac{4(N^3 - N)}{(4\pi)^3 \cdot 3^3} \int d^6x\sqrt{g_0}\ (2I_1 + I_2)\ +\ (4-\gamma)N\ \mathbf{A}_{tens.}\ . \tag{5.13}$$

Thus if the true value of γ is 3 instead of the naive value 1 (4.8) which follows directly from reduction of E_8 (4.3), ignoring possible F_4-dependent ($F_4^2R^3$) terms in the 11-d super-invariant $\mathcal{I}_2$, then $\mathbf{A}_{(2,0)}$ reproduces the scale anomaly (5.11) of a single (2,0) tensor multiplet. This $N = 1$ relation should be expected, given that a similar correspondence is true for the R-symmetry anomalies [11] (see below). Though we are unable to show that $\gamma = 3$ does follow from the $d = 7$ reduction of the 11-d super-invariant $\mathcal{I}_2$ containing P-odd $\mathcal{C}_3R^4$ term, we find it remarkable that the required value of γ differs from the naive value 1 simply by factor of 3.[23] [24]

Making a natural conjecture that the same relation $\mathcal{A}_{tens.} = (\mathcal{A}_{(2,0)})_{N=1}$ should be true between the full expressions for the conformal anomalies of the (2,0) theory and tensor

[23] In the original version of the present paper we mistakenly used the basis of type B invariants including I_3 of [8] instead of the correct invariant of [9] and as a result got the $O(N)$ term with extra coefficient 3, concluding that $\gamma = 1$ gives already the desired coefficient $4N^3 - 3N$ in (5.12).

[24] Note that if we were comparing the full local conformal anomalies evaluated for $R_{mn} = 0$ then, since the N^3 contribution (5.5) vanishes in this case, we would need $\gamma = \frac{3}{4}$ in order to reproduce the non-zero $R_{mn} = 0$ value of the tensor multiplet anomaly (5.8) by the $N = 1$ limit of the $O(N)$ term in (5.4).

multiplet, one can make a prediction about the complete structure of the $O(N)$ term in the (2,0) theory anomaly $\mathcal{A}_{(2,0)}$ (5.4) (cf. (5.5),(5.8))[25]

$$\mathcal{A}_{(2,0)} = -\frac{1}{(4\pi)^3 \cdot 3^2 \cdot 2^5}\Big[(4N^3 - \frac{9}{4}N)E_6 + (4N^3 - 3N)\cdot 8(12I_1 + 3I_2 - I_3) + O(\nabla_i J^i)\Big] , \tag{5.14}$$

or, equivalently,

$$\mathcal{A}_{(2,0)} = -\frac{N^3 - N}{(4\pi)^3 \cdot 3^2 \cdot 2^3}\Big[E_6 + 8(12I_1 + 3I_2 - I_3) + O(\nabla_i J^i)\Big] + N\mathcal{A}_{tens.} . \tag{5.15}$$

Using (5.7), we can rewrite (5.14) also as

$$\mathcal{A}_{(2,0)} = -\frac{N}{(4\pi)^3 \cdot 3 \cdot 2^7}\Big[E_6 + O(R_{mn}) + O(\nabla_i J^i)\Big] , \tag{5.16}$$

in agreement with the fact that for $R_{mn} = 0$ the conformal anomaly of the tensor multiplet becomes [9,36] $\mathcal{A}_{tens.} = -\frac{1}{(4\pi)^3 \cdot 3 \cdot 2^7}\big[E_6 + O(\nabla_i J^i)\big]$.

It is useful to compare the above expressions (5.12),(5.14) with the previously known results for the R-symmetry anomalies of the interacting (2,0) theory and free tensor multiplet theory. The 1-loop effective action Γ for a free 6-d tensor multiplet in a background of 6-d Lorentz curvature R and $SO(5)$ R-symmetry gauge field F has local $SO(6)$ and $SO(5)$ anomalies. They satisfy the descent relations $d(\delta\Gamma) = \delta I_7, \quad I_8 = dI_7$, with the 8-form anomaly polynomial I_8 being [13,14]

$$I_8^{tens.}(F,R) = \frac{1}{3 \cdot 2^4}\Big[p_2(F) - p_2(R) + \frac{1}{4}[p_1(F) - p_1(R)]^2\Big] , \tag{5.17}$$

with (here $F^2 \equiv F \wedge F$, etc.)

$$p_1(F) = \frac{1}{2} \text{ tr } \bar{F}^2 , \qquad p_2(F) = -\frac{1}{4}\Big(\text{tr } \bar{F}^4 - \frac{1}{2}\text{tr } \bar{F}^2 \wedge \text{tr } \bar{F}^2\Big) , \qquad \bar{F} = \frac{i}{2\pi} F . \tag{5.18}$$

[25] The shift of the coefficient of the E_6 term in the conformal anomaly seems to imply a contradiction between our assumption that the R^4 terms in the 11-d action (3.2) do not change the scale of $AdS_7 \times S^4$ solution (i.e. that the value of the 7-d action (4.5) evaluated on the AdS_7 solution is not changed), and the claim of [50] that the coefficient of the type A (Euler) term in the anomaly of a generic effective theory is determined only by the value of the action on the AdS solution.

The corresponding anomalies of the interacting (2,0) theory describing multiple M5 branes derived (by assuming that the total M5-brane anomaly + inflow anomaly should cancel) from the 11-d supergravity action (3.1) with the R^4 correction term (3.2) is [11]

$$I_8^{(2,0)}(F,R) = \frac{1}{3 \cdot 2^4} \Big[\ (2N^3 - N) \ p_2(F) \ - \ N \ p_2(R) \ + \ \frac{1}{4} N \ [p_1(F) - p_1(R)]^2 \Big] \ . \quad (5.19)$$

Here the $O(N^3)$ term comes [55] from the CS term in supergravity action (3.1) and the $O(N)$ term [12,14]– from the P-odd $\mathcal{C}_3 R^4$ part of the superinvariant $\mathcal{I}_2$ (3.2),(3.4). Equivalently,

$$I_8^{(2,0)}(F,R) = \frac{1}{3 \cdot 2^3} (N^3 - N) \ p_2(F) \quad + \quad N \ I_8^{tens.}(F,R) \ . \quad (5.20)$$

Thus for $N = 1$ the anomaly of the (2,0) theory is the same as the anomaly of a single tensor multiplet. This is the same type of a relation we have established above (cf. (5.13)) for the scale anomalies, with the crucial $O(N)$ contribution coming from the P-even E_8 part of the superinvariant $\mathcal{I}_2$ (3.4). This is obviously consistent with the fact that R-symmetry and conformal anomalies should be parts of the same 6-d supermultiplet.

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